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Conditions for mechanical Bloch oscillations

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Using the theory for surface waves, the propagation of a disturbance in elastic media is studied. An important and sufficient condition for achieving Bloch oscillations is the variation of the depth of the fluid. This variation produces changes in the refraction index and in the wave number of the disturbance in such a way that those changes exhibit discontinuities whose magnitudes are directly associated to the predicted frequency. A dispersion relation, that allows us to obtain the oscillations of the disturbance, is determined and applied. The calculation for stationary waves with such relation shows some indications suggesting oscillations which are a reproduction in the space domain of the Bloch oscillations for electrons under an electric field.

1 Introduction

Electronic Bloch oscillations were predicted 70 years ago, as a consequence of the translational periodic distribution of atoms in crystals under the influence of an electric field. However, the experimental confirmation had to wait until artificial periodic structures were grown in the laboratory. As a result of these achievements, photonic and atomic Bloch oscillations have also been found [1-4]. All these phenomena, in intrinsically different systems, show common features that suggest the extension of these regularities to mechanical systems, as has been discussed in ref. [5].

The main idea consists in trying to find a unified description for all these systems. Due to the quantum nature of the known Bloch oscillations, the goal of the present work is to describe a classical system by using formal principles of physics and interpret the results in terms of similarities with the quantum systems.

First, we establish the conditions that make it possible to observe the Mechanical Bloch Oscillations (MBO) by using the theory of Hydrodynamics applied to the proposed experimental design [6].

Next, after choosing a dispersion relation for the mechanical system, we proceed to find the relevant parameters to detect MBO; finally the results of our related calculations are discussed.

2 The system

The extension of the optic-electronic analogy has been proposed for elastic wave systems propagating in periodic structures [5] within the framework of Hydrodynamics. An elastic medium such as a liquid, whose surface suffers a point perturbation, exhibits the propagation of it, determined by the capillarity characteristics of the liquid, the gravitational interaction, the type of the perturbation and the geometry of the recipient.

A two-dimensional analog of a plane wave can be generated in a ripple tank by a flat board that oscillates up and down in the water to produce wave-fronts that are straight lines.

The classical stationary wave equation is used to describe the perturbation, which can be written as

$$(\nabla^2 + \frac{w^2}{V^2})\Psi(x, y) = 0 \tag{1}$$

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that is known as the Helmholtz equation, where *w* is the angular frequency and *V* the phase velocity of the propagation, which is related to the dispersion relation of the system [6].

For fluids with negligible viscosity the dispersion relation depends on the height of the fluid, capillarity effects and gravitational interaction where ***K*** is the wave vector, *T* the surface tension and ρ the density of the liquid

$$w = \sqrt{gK\left(1 + \frac{TK^2}{g\rho}\right)\tanh(hK)} \tag{2}$$

This relation can be studied in several limits taking into account the parameters of perturbation and the boundary conditions. We used the surface wave limit of gravity in which the capillarity effects are negligible due to the fact that the wavelength is larger than the depth of the fluid [7].

Under these conditions the dispersion relation is reduced to consider only non-dispersive waves and the depth *h* of the medium is a function of the local point where the field Ψ is calculated.

$$\frac{w^2}{K^2} = g \quad h(x,y) \tag{3}$$

Under these conditions the propagation velocity and the refraction index depend on $h^{1/2}$, making the depth of the fluid the relevant parameter to control the functional dependence of the wave vector through the surface of the elastic medium

By introducing (3) in (1), and applying variable separation for free ends boundary conditions, is obtained for the deformation Ψ the expression

$$\Psi(x,y) = A \quad Cos(K_y y) \quad X(x) \tag{4}$$

where A is the amplitude of the perturbation and X(x) the solution of the differential equation in the direction in which the deep is changing, as we are going to discuss in the next section.

3 The main characteristics of the propagation medium

In order to create a Wannier ladder in the wave vector, we choose the dependence of the refraction index as a discontinuously increasing function on the depth *h* [5, 8]. The depth varies in *x* direction while *y* is a constant. Searching a linear increasing of the refraction index with the position, an adequate function fulfilling these requirements is

$$h_i(x,y) = \frac{\beta}{(x+\alpha_i)^2} \tag{5}$$

where the parameters α and β are constants that define the geometry of the bottom of the recipient which contains the liquid; β is associated with the change of depth per unit length, while α shifts the origin along the *y* axis, it generates discontinuities in ***K*** to define the Wannier Ladder [9].

The designed recipient requires several regions with two different depth functions, as given in equation (5) with different values of α along the *x* axis.

The initial conditions are determined by the length in the *y* axis and the smoothness of the field Ψ at the boundaries between different depth regions. So, the field and its derivative should be continuous at the interfaces, and then the solution of the differential equation for *X(x)* for the chosen conditions is

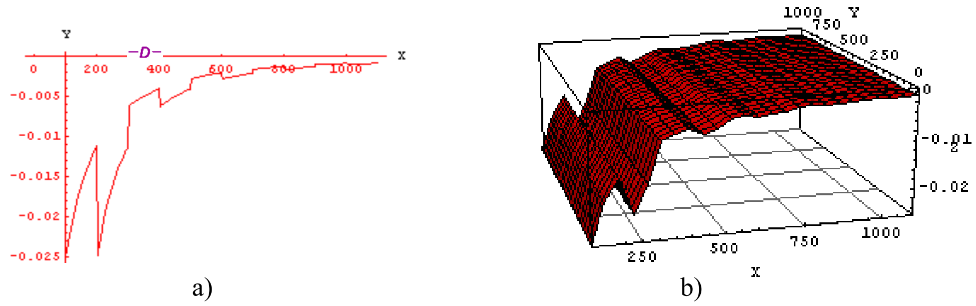
$$X(x) = t_1 \; e^{\frac{-i(bx+cx^2)}{2\sqrt{c}}} \; H[n,\tau] \; + \; t_2 \; e^{\frac{-i(bx+cx^2)}{2\sqrt{c}}} \; F_1[m,\frac{1}{2},\tau] \tag{6}$$

$$n = \frac{ib^2 + i4ac - 4c^{3/2}}{8c^{3/2}} \qquad m = \frac{ib^2 + i4ac - 4c^{3/2}}{16c^{3/2}} \qquad \tau = \frac{(-1)^{1/4} b}{2c^{3/4}} + (-1)^{1/4} c^{1/4} x$$

where H[n,τ] is the nth order Hermite polynomial and F_1[m,1/2,τ] is the mth order confluent hyper geometric function. The constants *a, b* and *c* are determined by the geometric parameters α and β, the frequency *w*, the *K* vector in direction *y* and the gravity *g*, following

$$a_i = K_y^{\;2} - \frac{w^2\alpha_i}{g\beta} \qquad b_i = \frac{2\alpha_i w^2}{g\beta} \qquad c = \frac{w^2}{g\beta} \tag{7}$$

while t_1 and t_2 are the integration constants, which should be different for each region in x direction due to the alternating conditions.
The studied system is a liquid confined in a square boundary and variable depth h container. Figure 1 shows the lateral profile and the perspective view of the designed recipient.



a) b)

Fig. 1 a) Lateral profile of the recipient. b) View of the bottom of the recipient.

Following the extension proposed in [5], and the deep function discussed above the space period of the MBO oscillations is given by

$$P_{MBO} = \frac{2\pi\sqrt{g\beta}}{wD} \tag{8}$$

where is remarkable the inverse linear dependence on D (equivalent to the lattice constant) and the direct dependence on the square root of β, that in analogy with the electronic case, is playing the important role of the external field to define the period of the Bloch-like oscillations.

4 Results and discussion

In order to test the model, we propose the design of an ripple tank filled with a liquid, which is confined within a volume of square surface of 1000 m^2 and variable depth h with following parameters: $w=\pi/10$ s^{-1}, $K_y = 0.062$ m^{-1}, $\alpha_1 = 0$ m, $\alpha_2 = 20$.
As an illustration of our results we present following features. In Figs. 2(a), 2(b) and 2(c) is shown one main fringe, which depends on the variation of β and D and accounts for the predicted MBO space period in (8). Moreover, also the numerical value for the width of the fringe agrees very well with the predicted value. Since this is the typical localization feature for the Bloch oscillations, we proceed to confirm our proposal by testing the dependence of the fringe width on the external field (β) and the lattice constant (D) choosing following parameters; related to Fig. 2(a), in Fig. 2(b) with double value of D, and in Fig. 2(c), with β divided by ten, so its square root is approximately reduced three times. The suppression of the external field is illustrated in Fig. 2(d) for the flat bottom case ($\beta = 0$), in complete analogy with a simple superlattice periodic potential ($E = 0$) in electronic systems. This agrees with reference [9], where is shown for elastic disturbances that the bottom design influences the amplitude of the stationary wave and forms fringes associated with transmission band gaps.
Extending language from electronic Bloch oscillations to this classical system, the main fringe in Figs. 2(a),(b) and (c) can be interpreted in terms of uncertainty principle, considering that x resolution, that is to say, the capability to observe the initial disturbance features, is limited for discontinuities (indetermination) in K_x.
In conclusion, we found conditions to form Wannier Ladder and the related MBO in a mechanical system, and report strong indications for the existence of Bloch-like oscillations in a system such as a ripple tank, with a carefully designed bottom, given by a surface that oscillates up and down on the liquid showing even the localization of the oscillations as an external field, squared root of α is applied.

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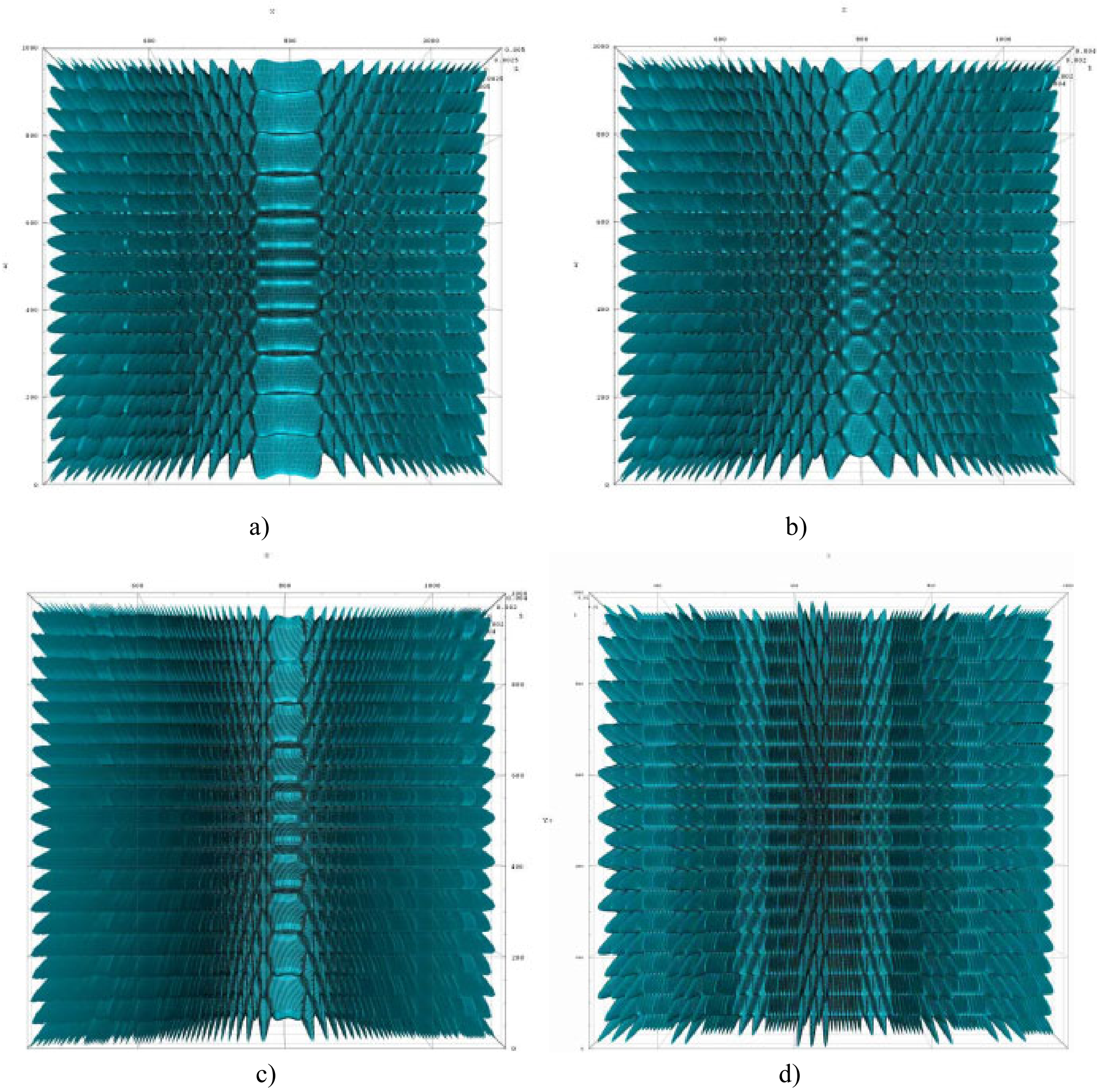

Fig. 2 Field Ψ calculated for the ripple tank with: a) $\beta = 2250$, $D = 50$; b) $\beta = 2250$, $D = 100$; c) $\beta = 225$, $D = 50$; and d) alternating flat bottom, $D = 50$.

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