Coupling, Relaxation and Coherent Emission in Semiconductor Double Quantum Dots

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Abstract. We studied the dynamical response of cylindrical and spherical double quantum dots to an external time-dependent electric field pulse. The density matrix evolution is calculated in the constant dissipation operator approach, considering electron-phonon and electron-electron interaction. This dynamics is evaluated in the high delocalization regime, where the process of decoherence is dominated by relaxation rates. Such rates and their related decay times are obtained from a Fermi golden rule treatment of the interactions. We found convenient conditions for coherent emission in the cylindrical dots case, while for spherical dots the fast relaxation from excited levels destroys the coherence in a time shorter than the period of any of the involved quantum beat oscillations.

Keywords: Coupled Quantum Dots, Relaxation, Coherent Emission.

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INTRODUCTION

Double quantum dots arrays are good candidates to be useful in the implementation of nano-devices both for optoelectronics and quantum computation. Tunability of states and long coherence times are the main advantages of such systems1,2. In a previous work, we introduced a model to evaluate numerically the dynamics of a single carrier in a rectangular double quantum dot, focusing in the first three levels3. Here we basically use the same treatment but extended to five levels because of the geometry of the considered dots. In the first section, we found the electronic levels for the studied geometries. After that, we use the density matrix formalism to evaluate the evolution of the system after the application of an external pulse. Finally, using our computed decay rates, we calculate the total dipole moment looking for the viability of coherent emission from the double dot-arrays.

ENERGY STATES AND COUPLING

The systems we focus in are pairs of quantum dots close enough to be electronically coupled, which is equivalent to having their individual states hybridized. They are chosen as islands of GaAs embedded in a Al0.4Ga0.6As matrix. Experimental parameters are as in reference 3. The confinement potential is modeled as a finite potential step. We consider spherical and cylindrical geometries. The case with cylindrical shape has two options; stacked or lateral coupling. These two configurations are expected to have different behavior since in the latter one, the axial symmetry is broken. We found the energy levels by solving numerically the modified Schrödinger equation

$$-\frac{\hbar^2}{2m^*} \nabla \cdot \left( \frac{1}{m^*} \nabla \right) + V(x, y, z) - eFx$$

where $m^*$ is the electron effective mass, $V$ the offset in the conduction band of the two materials, $e$ is the elemental charge, $F$ is the bias electric field applied in the $x$ direction (the coupling direction), $\Psi$ is the one-electron envelope function and $E$ the proper energy. The bias field is used as coupling parameter to tune the High Delocalization Regime (HDR). A volume ratio of ~4.3 between the dots of the pair was established as suitable to reach the HDR when the smaller dots are around 300nm3.

TEMPORAL RESPONSE

We focus on the first two energy values of individual spherical dots and at the first three of cylindrical dots. The spherical dots have non-degenerate ground eigenenergy and a three-fold degenerate first excited eigenenergy. The cylindrical
dots also have a non-degenerate ground level and first excited eigenenergy, but a two-fold degenerate second excited energy value close to the former one. So, in both cases, to work with coupled dots with these levels hybridized it is necessary to consider a five-level system: one associated with the smaller dot and four with the bigger one.

The occupation probability of energy levels and the coherence between states for the reduced five-level system after the application of an ultrafast electric field pulse (amplitude 50 mV/nm, duration 4 fs, in our calculations), can be obtained from the density matrix, whose evolution is given by the Liouville equation, which in the Lindblad form in the constant dissipation operator approximation is

\[
\dot{\rho}_{ij} = -\frac{i}{\hbar} [H_{ij}^U(t), \rho_{ij}(t)] - i\hbar \Gamma_{ij}.
\]  

where \( \rho_{ij} \) is the \( ij \) density matrix component, \( H_{ij}^U \) the matrix element of the hamiltonian including the exciting electric field, and \( \Gamma_{ij} \) are the relaxation rates to be mentioned at the next section.

**RELAXATION RATES**

Since we are tuning the system to the HDR, we neglect the pure dephasing rates and link the decoherence process only to the relaxation times. We consider interactions of the confined electron with no confined ones present in the matrix as conduction carriers, and with the lattice through acoustic phonons. Their respective scattering rates are evaluated by the Fermi golden rule approximation

\[
\Gamma_{ij} = \frac{2\pi}{\hbar} |H_{ij}^f|^2 \delta(E),
\]

where \( H_{ij}^f \) is the hamiltonian of the correspondent interaction and \( \delta(E) \) guarantees energy conservation.

**RESULTS AND CONCLUSION**

Figure 1 shows the Total Dipole Moment from the excited levels of the density matrix for the three considered geometries. An oscillating dipole moment means coherent emission and for cylindrically shaped dots, especially for the ones coupled vertically, the observation of such emission is feasible since many cycles are completed before the inhibition by scattering by phonons and conduction electrons. The spherical geometry was found not to be suitable to observe coherent emission since the attenuation rate is bigger than the emission frequencies.

![FIGURE 1](image-url) 

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